

Noncommutative Quantum Gravity

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Abstract

We discuss the BRST and anti-BRST symmetries for perturbative quantum gravity in noncommutative spacetime. In this noncommutative perturbative quantum gravity the sum of the classical Lagrangian density with a gauge fixing term and a ghost term is shown to be invariant the noncommutative BRST and the noncommutative anti-BRST transformations. We analyse the gauge fixing term and the ghost term in both linear as well as non-linear gauges. We also discuss the unitarity evolution of the theory and analyse the violation of unitarity of by introduction of a bare mass term in the noncommutative BRST and the noncommutative anti-BRST transformations.

Key words: Noncommutative BRST, Noncommutative Anti-BRST
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1 Introduction

Noncommutative spacetime was originally studied as a mechanism for providing a natural cut to control ultraviolet divergences [1]. However this motivation to study noncommutative field theory ended with the success of the renormalization theory. The real motivation behind the study of noncommutative field theory is its close relation to the string theory [2]. The presence of an anti-symmetric tensor background along the D -brane world volumes is an important source for noncommutativity in string theory [3, 4].

The relation between noncommutative field theories and quantum gravity has been studied by many authors [5, 6, 7, 8]. The relation between noncommutative gravity and the cosmological constant has been examined [9]. In doing so it was concluded that noncommutative gravity leads to the existence of a tiny non zero cosmological constant of the order of the square of the Hubble constant. Black holes in noncommutative gravity has also been thoroughly studied [10, 11, 12]. Even in noncommutative gravity there is no correlation between the different modes of radiation and so information does not come out continuously during the evaporation process. However, due to spacetime noncommutativity, information might be preserved by a stable black hole remnant [13].

Perturbative quantum gravity on noncommutative flat spacetime has also been discussed [14]. In this paper will will discuss the BRST and anti-BRST symmetries for this noncommutative perturbative quantum gravity. It may be be remarked that the BRST symmetry for noncommutative Yang-Mills theories

has already been analysed [15, 16, 17, 18]. The BRST symmetry for spontaneously broken gauge theories in noncommutative spacetime has also been discussed [19]. In case of noncommutative gauge theory the Hilbert space of physical states is determined by the cohomology space of the BRST operator as in the commutative case. The anti-BRST symmetry for noncommutative Yang-Mills theories has also been analysed [20].

The BRST and the anti-BRST symmetries for commutative perturbative quantum gravity in flat spacetime have been studied by a number of authors [21, 22, 23] and their work has been summarized by N. Nakanishi and I. Ojima [24]. The BRST symmetry in two dimensional curved spacetime has been thoroughly studied [25, 26, 27]. The BRST and the anti-BRST symmetries for topological quantum gravity in curved spacetime have also been studied [28, 29]. All this work has been done in linear gauges. The BRST and anti-BRST symmetries for perturbative quantum gravity in non-linear gauges has also been analysed [30]. This analysis was done in arbitrary dimensional curved spacetime.

In this paper we shall study the BRST and the anti-BRST symmetries for noncommutative perturbative quantum gravity in both linear and non-linear gauges. Furthermore, it will be demonstrated that the addition of a bare mass term violates the nilpotency of the noncommutative BRST and the noncommutative anti-BRST transformations and this in turn violates unitarity of the resultant theory.

2 Linear Gauges

In perturbative gravity one splits the full metric $g_{ab}^{(f)}$ into the metric for the background spacetime and a small perturbation around it. The covariant derivatives along with the lowering and raising of indices are compatible with the metric for the background spacetime. The small perturbation is viewed as the field that is to be quantized. For simplicity we shall take our background spacetime to be flat. Then noncommutativity is introduced by replacing the all the fields by noncommutative fields and all the product of fields by the Moyal $*$ -product. The Moyal $*$ -product of two noncommutative fields say h^{ab} and h_{ab} is defined as follows [2]

$$h^{ab} * h_{ab}(x) = \exp\left(\frac{i}{2}\theta^{ab}\partial_a\partial_b\right) h^{ab}(x)h_{ab}(x+\epsilon)|_{\epsilon=0}. \quad (1)$$

The Lagrangian density for pure gravity on noncommutative spacetime is given by [14]

$$\mathcal{L}_c = \sqrt{g}^{(f)} * R^{(f)}, \quad (2)$$

where we have adopted units such that $16\pi G = 1$. Here $R^{(f)} = g^{ab} * R_{ab}^{(f)}$, where

$$R_{ab}^{(f)} = R^{c(f)}{}_{acb} = \partial_c \Gamma_{bc}^{c(f)} - \partial_b \Gamma_{ca}^{c(f)} + \Gamma_{cd}^{c(f)} * \Gamma_{ba}^{d(f)} - \Gamma_{bd}^{c(f)} * \Gamma_{ca}^{d(f)}. \quad (3)$$

Here $R_{abc}^{d(f)}$ satisfies the Bianchi identity

$$\partial_e R_{abc}^{d(f)} + \partial_c R_{aeb}^{d(f)} + \partial_b R_{ace}^{d(f)} = 0. \quad (4)$$

We can expand $g_{ab}^{(f)}$ as follows

$$g_{ab}^{(f)} = \eta_{ab} + h_{ab}. \quad (5)$$

The expansion of the Ricci scalar $R^{(f)}$ and the metric $g_{ab}^{(f)}$ in terms of the Minkowski metric η_{ab} and a small perturbation around it h_{ab} generates the Lagrangian density for noncommutative perturbative quantum gravity. There are infinitely many terms in the Lagrangian for this noncommutative perturbative quantum gravity.

All the degrees of freedom in h_{ab} are not physical. This is because the Lagrangian density for it is invariant under a gauge transformation,

$$\delta_\Lambda h_{ab} = \partial_a \Lambda_b + \partial_b \Lambda_a + \mathcal{L}_{(\Lambda)} h_{ab}, \quad (6)$$

where

$$\mathcal{L}_{(\Lambda)} h_{ab} = \Lambda^c * \partial_c h_{ab} + h_{ac} * \partial_b \Lambda^c + h_{cb} * \partial_a \Lambda^c. \quad (7)$$

These unphysical degrees of freedom will give rise to constraints [31] in the canonical quantization and divergences in the partition function [32] in the path integral quantization. So before we can quantize this theory, we need to remove these unphysical degrees of freedom. This can be done by the addition of a noncommutative gauge fixing term and a noncommutative ghost term. The sum of a gauge fixing term and a ghost term can now be written as

$$\begin{aligned} \mathcal{L}_g &= -\frac{i}{2} s \bar{s} (h^{ab} * h_{ab}) + \frac{i\alpha}{2} \bar{s} (b^a * c_a) \\ &= \frac{i}{2} \bar{s} s (h^{ab} * h_{ab}) - \frac{i\alpha}{2} s (b^a * \bar{c}_a), \end{aligned} \quad (8)$$

where s denotes the BRST transformations which is given by

$$\begin{aligned} s h_{ab} &= \partial_a c_b + \partial_b c_a + \mathcal{L}_{(c)} h_{ab}, \\ s c^a &= -c_b * \partial^b c^a, \\ s \bar{c}^a &= b^a, \\ s b^a &= 0, \end{aligned} \quad (9)$$

and \bar{s} denotes the anti-BRST transformations which is given by

$$\begin{aligned} \bar{s} h_{ab} &= \partial_a \bar{c}_b + \partial_b \bar{c}_a + \mathcal{L}_{(\bar{c})} h_{ab}, \\ \bar{s} c^a &= -b^a - 2\bar{c}_b * \partial^b c^a, \\ \bar{s} \bar{c}^a &= -\bar{c}_b * \partial^b \bar{c}^a, \\ \bar{s} b^a &= -b^b * \partial_b c^a. \end{aligned} \quad (10)$$

Here $\mathcal{L}_{(c)}$ and $\mathcal{L}_{(\bar{c})}$ are given by

$$\begin{aligned} \mathcal{L}_{(c)} h_{ab} &= c^c * \partial_c h_{ab} + h_{ac} * \partial_b c^c + h_{cb} * \partial_a c^c, \\ \mathcal{L}_{(\bar{c})} h_{ab} &= \bar{c}^c * \partial_c h_{ab} + h_{ac} * \partial_b \bar{c}^c + h_{cb} * \partial_a \bar{c}^c. \end{aligned} \quad (11)$$

Now using $c^a * c_a = \bar{c}^a * \bar{c}_a = 0$ and the Bianchi identity, we can show after a straightforward calculation that these transformations are nilpotent,

$$s^2 = \bar{s}^2 = 0. \quad (12)$$

In fact, they can also be shown to satisfy $\bar{s}s + s\bar{s} = 0$. Now the effective Lagrangian density which is given by the sum of the original classical Lagrangian

density with these gauge fixing and ghost terms is invariant under these non-commutative BRST and noncommutative anti-BRST transformations.

$$s \mathcal{L}_{eff} = \bar{s} \mathcal{L}_{eff} = 0, \quad (13)$$

where

$$\mathcal{L}_{eff} = \mathcal{L}_c + \mathcal{L}_g. \quad (14)$$

It is so because for the original classical Lagrangian the noncommutative BRST or the anti-BRST noncommutative transformations are only gauge transformations with the gauge parameter replaced by ghosts or the anti-ghosts. Furthermore, as the sum of the gauge fixing and the ghost term can be written as a total noncommutative BRST or a total noncommutative anti-BRST variation and both these transformations are nilpotent, the Lagrangian is invariant under them. It is obvious the in Landau gauge as $\alpha = 0$, we can express the gauge fixing Lagrangian density as a combination of total noncommutative BRST and total noncommutative anti-BRST variations

$$\begin{aligned} \mathcal{L}_g &= -\frac{i}{2} s \bar{s} (h^{ab} * h_{ab}) \\ &= \frac{i}{2} \bar{s} s (h^{ab} * h_{ab}). \end{aligned} \quad (15)$$

3 Non-Linear Gauges

It is known that for perturbative quantum gravity in Curci-Ferrari gauge we can write the sum of the gauge fixing term and the ghost term as a combination of total BRST and total anti-BRST variations for any value of α [30]. We will show here that this can also be done for noncommutative perturbative quantum gravity. The noncommutative BRST transformations for noncommutative perturbative quantum gravity in Curci-Ferrari gauge can be written as

$$\begin{aligned} s h_{ab} &= \partial_a c_b + \partial_b c_a + \mathcal{L}_{(c)} h_{ab}, \\ s c^a &= -c_b * \partial^b c^a, \\ s \bar{c}^a &= b^a - \bar{c}^b * \partial_b c^a, \\ s b^a &= -b^b * \partial_b c^a - \bar{c}^b * \partial_b c^d * \partial_d c^a, \end{aligned} \quad (16)$$

and the noncommutative anti-BRST transformation for noncommutative perturbative quantum gravity in Curci-Ferrari gauge can be written as

$$\begin{aligned} \bar{s} h_{ab} &= \partial_a \bar{c}_b + \partial_b \bar{c}_a + \mathcal{L}_{(\bar{c})} h_{ab}, \\ \bar{s} \bar{c}^a &= -\bar{c}_b * \partial^b \bar{c}^a, \\ \bar{s} c^a &= -b^a - \bar{c}^b * \partial_b c^a, \\ \bar{s} b^a &= -b^b * \partial_b \bar{c}^a + c^b * \partial_b \bar{c}^d * \partial_d \bar{c}^a. \end{aligned} \quad (17)$$

Now using $c^a * c_a = \bar{c}^a * \bar{c}_a = 0$ and the Bianchi identity, we can show after a straightforward but lengthy calculation that these transformations are nilpotent,

$$s^2 = \bar{s}^2 = 0. \quad (18)$$

In fact, they also satisfy $\bar{s}s + s\bar{s} = 0$. We can now write a gauge fixing term and the ghost term as a combination of a total BRST and a total anti-BRST variation, as

$$\begin{aligned}\mathcal{L}_g &= \frac{i}{2} s\bar{s} [h^{ab} * h_{ab} - i\alpha \bar{c}^a * c_a] \\ &= \frac{-i}{2} \bar{s}s [h^{ab} * h_{ab} - i\alpha \bar{c}^a * c_a].\end{aligned}\quad (19)$$

Now we can analyse the effect of the addition of a bare mass term to this Lagrangian density. The Lagrangian density for the noncommutative perturbative quantum gravity in massive Curci-Ferrari gauge can be written as

$$\begin{aligned}\mathcal{L}_g &= \frac{i}{2} [s\bar{s} - im^2] [h^{ab} * h_{ab} - i\alpha \bar{c}^a * c_a] \\ &= \frac{i}{2} [-\bar{s}s - im^2] [h^{ab} * h_{ab} - i\alpha \bar{c}^a * c_a],\end{aligned}\quad (20)$$

where the noncommutative BRST transformations are given by

$$\begin{aligned}s h_{ab} &= \partial_a c_b + \partial_b c_a + \mathcal{L}_{(c)} h_{ab}, \\ s c^a &= -c_b * \partial^b c^a, \\ s \bar{c}^a &= b^a - \bar{c}^b * \partial_b c^a, \\ s b^a &= im^2 c^a - b^b * \partial_b c^a - \bar{c}^b * \partial_b c^d * \partial_d c^a,\end{aligned}\quad (21)$$

and the noncommutative anti-BRST transformations are given by

$$\begin{aligned}\bar{s} h_{ab} &= \partial_a \bar{c}_b + \partial_b \bar{c}_a + \mathcal{L}_{(\bar{c})} h_{ab}, \\ \bar{s} \bar{c}^a &= -\bar{c}_b * \partial^b \bar{c}^a, \\ \bar{s} c^a &= -b^a - \bar{c}^b * \partial_b c^a, \\ \bar{s} b^a &= im^2 \bar{c}^a - b^b * \partial_b \bar{c}^a + c^b * \partial_b \bar{c}^d * \partial_d \bar{c}^a.\end{aligned}\quad (22)$$

The addition of bare mass term breaks the nilpotency of these noncommutative BRST and anti-BRST transformations. The BRST and the anti-BRST transformations now satisfy

$$s^2 = \bar{s}^2 \sim im^2. \quad (23)$$

However, the nilpotency of the BRST and the anti-BRST transformations is restored in the zero mass limit.

It may be noted that the Lagrangian density for sum of the ghost term and gauge fixing term in the Landau gauge and these non-linear gauges can be expressed as a combination of total BRST and total anti-BRST variations. So the effective Lagrangian density in these gauges is also invariant under a symmetry called the *FP*-conjugation in these gauges. This *FP*-conjugation is given by

$$\begin{aligned}\delta_{FP} h_{ab} &= 0, \\ \delta_{FP} c^a &= \bar{c}^a, \\ \delta_{FP} \bar{c}^a &= -c^a, \\ \delta_{FP} b^a &= b^a - 2c^b * \partial_b c^a.\end{aligned}\quad (24)$$

Even though the nilpotency is violated in the massive Curci-Ferrari gauge, the *FP*-conjugation is maintained. The invariance of ordinary Yang-Mills theories under *FP*-conjugation for these gauges is well known [35, 36, 37, 38].

4 Conserved Charges

Conserved charges can only be defined properly for spacelike noncommutativity. This is because if we impose the full spacetime noncommutativity, we will end up having higher order time derivatives in our theory. This will spoil the unitarity of the theory. Thus, we will now restrict our discussions to spacelike noncommutativity. So, we will set $\theta^{i0} = 0$. In ordinary field theories there exists a divergenceless current corresponding to each symmetry transformation. In noncommutative field theories the divergence of the corresponding current does not vanish. It is rather equal to the Moyal bracket of some functions [33]. However, this Moyal bracket vanishes for the spacelike noncommutativity, when it is integrated over all spatial coordinates [34]. Hence, again a conserved charge can be associated with a symmetry transformation. This conserved charge commutes with the Hamiltonian of the theory. Let $f(x)$ and $g(x)$ be two local functions in a noncommutative spacetime. Now the divergence of the current associated with a symmetry in this spacetime, can be written as

$$[f(x), g(x)]_* = \partial^a J_a, \quad (25)$$

where we have defined the bracket $[f(x), g(x)]_*$ as

$$[f(x), g(x)]_* = f(x) * g(x) - g(x) * f(x). \quad (26)$$

In the case of spacelike noncommutativity, we have $\theta^{i0} = 0$, and so we get

$$\int d^3x [f(x), g(x)]_* = 0. \quad (27)$$

So, we can write the conserved charge corresponding to the symmetry as

$$Q = \int d^3x J^0. \quad (28)$$

The total Lagrangian which is given by the sum of the original Lagrangian of noncommutative perturbative quantum gravity, the gauge fixing term and the ghost term is invariant under the noncommutative BRST and the noncommutative anti-BRST transformations. We can calculate the conserved charges corresponding to the invariance of this total Lagrangian of noncommutative perturbative quantum gravity under these transformations. To do so, we first calculate the currents associated with these transformations,

$$\begin{aligned} J_a^{(B)}(x) &= \frac{\partial \mathcal{L}_{eff}}{\partial h_{ab}} * s h_{ab} + \frac{\partial \mathcal{L}_{eff}}{\partial c_a} * s c_a + \\ &\quad \frac{\partial \mathcal{L}_{eff}}{\partial \bar{c}_a} * s \bar{c}_a + \frac{\partial \mathcal{L}_{eff}}{\partial b_a} * s b_a, \\ \bar{J}_a^{(B)}(x) &= \frac{\partial \mathcal{L}_{eff}}{\partial h_{ab}} * \bar{s} h_{ab} + \frac{\partial \mathcal{L}_{eff}}{\partial c_a} * \bar{s} c_a + \\ &\quad \frac{\partial \mathcal{L}_{eff}}{\partial \bar{c}_a} * \bar{s} \bar{c}_a + \frac{\partial \mathcal{L}_{eff}}{\partial b_a} * \bar{s} b_a. \end{aligned} \quad (29)$$

Here J_a is the current associated with the noncommutative BRST symmetry and \bar{J}_a is the current associated with noncommutative anti-BRST symmetry.

Now we can calculate the BRST charge Q_B and anti-BRST charge \bar{Q}_B associated with the currents J_a and \bar{J} as follows,

$$\begin{aligned} Q_B &= \int d^3x J_{(B)}^0, \\ \bar{Q}_B &= \int d^3x \bar{J}_{(B)}^0. \end{aligned} \quad (30)$$

We can also define a conserved current $J_a^{(FP)}$ corresponding to FP -conjugation as

$$\begin{aligned} J_a^{(FP)}(x) &= \frac{\partial \mathcal{L}_{eff}}{\partial h_{ab}} * \delta_{FP} h_{ab} + \frac{\partial \mathcal{L}_{eff}}{\partial c_a} * \delta_{FP} c_a + \\ &\quad \frac{\partial \mathcal{L}_{eff}}{\partial \bar{c}_a} * \delta_{FP} \bar{c}_a + \frac{\partial \mathcal{L}_{eff}}{\partial b_a} * \delta_{FP} b_a, \end{aligned} \quad (31)$$

and the conserved charge corresponding to it as

$$Q_{FP} = \int d^3x J_{(FP)}^0. \quad (32)$$

In noncommutative perturbative gravity the BRST and the anti-BRST charges are nilpotent for all gauges except the massive Curci-Ferrari gauge. So, we will first restrict our discussion to gauges other than the massive Curci-Ferrari gauge. Then, we will analyse the effect of having the massive Curci-Ferrari gauge. As we have restricted our discussion to gauges other than the massive Curci-Ferrari gauge, so for any state $|\phi\rangle$, we have

$$\begin{aligned} Q_B^2 |\phi\rangle &= 0, \\ \bar{Q}_B^2 |\phi\rangle &= 0. \end{aligned} \quad (33)$$

We now define the physical states $|\phi_p\rangle$ as the states annihilated by the BRST charge

$$Q_B |\phi_p\rangle = 0. \quad (34)$$

We will obtain the same result if we define the physical states as the states annihilated by the anti-BRST charge

$$\bar{Q}_B |\phi_p\rangle = 0. \quad (35)$$

We will get the same physical result by using either of these definitions of the physical states. The physical states that are obtained from other states by the action of either the BRST or the anti-BRST charges, are orthogonal to all physical states. They are even orthogonal to themselves. Thus, two physical states that differ from each other by such a state will be indistinguishable. Let the asymptotic physical states be

$$\begin{aligned} |\phi_{pa,out}\rangle &= |\phi_{pa}, t \rightarrow \infty\rangle, \\ |\phi_{pb,in}\rangle &= |\phi_{pb}, t \rightarrow -\infty\rangle. \end{aligned} \quad (36)$$

Now a \mathcal{S} -matrix element can be written as

$$\langle \phi_{pa,out} | \phi_{pb,in} \rangle = \langle \phi_{pa} | \mathcal{S}^\dagger | \phi_{pb} \rangle. \quad (37)$$

The BRST and the anti-BRST charges commute with the Hamiltonian because they are conserved charges. So, the time evolution of any physical state will also be annihilated by them.

$$\begin{aligned} Q_B \mathcal{S}|\phi_{pb}\rangle &= 0, \\ \overline{Q}_B \mathcal{S}|\phi_{pb}\rangle &= 0 \end{aligned} \quad (38)$$

Thus, the states $\mathcal{S}|\phi_{pb}\rangle$ can only be a linear combination of physical states,

$$\langle\phi_{pa}|\mathcal{S}^\dagger\mathcal{S}|\phi_{pb}\rangle = \sum_i \langle\phi_{pa}|\mathcal{S}^\dagger|\phi_{0,i}\rangle\langle\phi_{0,i}|\mathcal{S}|\phi_{pb}\rangle. \quad (39)$$

Since the full \mathcal{S} -matrix is unitary this relation implies that the S -matrix restricted to physical sub-space is also unitarity. It may be noted that the nilpotency of the BRST and the anti-BRST charges was essential for proving the unitarity of the \mathcal{S} -matrix. Now as the BRST and the anti-BRST charges are not nilpotent in the massive Curci-Ferrari gauge,

$$\begin{aligned} Q_B^2|\phi\rangle &\neq 0, \\ \overline{Q}_B^2|\phi\rangle &\neq 0. \end{aligned} \quad (40)$$

So the above argument does not hold for the massive Curci-Ferrari gauge. Thus, the \mathcal{S} does not factorize in the massive Curci-Ferrari gauge

$$\langle\phi_{pa}|\mathcal{S}^\dagger\mathcal{S}|\phi_{pb}\rangle \neq \sum_i \langle\phi_{pa}|\mathcal{S}^\dagger|\phi_{0,i}\rangle\langle\phi_{0,i}|\mathcal{S}|\phi_{pb}\rangle, \quad (41)$$

and the resultant theory is not unitarity. However, the nilpotency of the BRST and the anti-BRST charges is restored in the zero mass limit. As the unitarity of the theory depended on the nilpotency of these charges, so the unitarity is also restored in the zero mass limit. This loss of unitarity can have important physical consequences as it is expected that certain quantum gravitational process might lead to the violation of unitarity [39].

5 Conclusion

In this paper we have generalized certain results of perturbative quantum gravity to noncommutative spacetime. We have expressed the gauge fixing Lagrangian density for perturbative quantum gravity as a combination of total BRST and total anti-BRST variations in Landau gauge and non-linear gauges. We have shown that the nilpotency of the noncommutative BRST and the noncommutative anti-BRST leads to the unitarity of theory. Furthermore, as the addition of a bare mass term violated the nilpotency of the noncommutative BRST and the noncommutative anti-BRST transformations, so the unitarity of noncommutative perturbative quantum gravity is also violated in the massive Curci-Ferrari gauge. This may have important physical consequences for those processes in quantum gravity where the unitarity could be violated [39]. All these results were already known to hold for commutative perturbative quantum gravity and we have shown here that they also hold for noncommutative perturbative quantum gravity.

The BRST and the anti-BRST symmetries for the noncommutative Yang-Mills theories have only been studied in linear gauges. So it will be interesting to analyse the BRST and the anti-BRST symmetries for the noncommutative Yang-Mills theories in non-linear gauges. It is known for ordinary Yang-Mills theories in the Landau and Curci-Ferrari gauges that the BRST and the anti-BRST are generators are part of a larger $SL(2, R)$ algebra known as the Nakanishi-Ojima algebra [35]. It will be interesting to find out if this algebra also holds for noncommutative Yang-Mills theories and noncommutative perturbative quantum gravity. Furthermore, it is known for ordinary Yang-Mills theories that this algebra is broken by ghost condensation [38]. It is highly likely that a similar thing occurs in noncommutative Yang-Mills theories. However, the study of ghost condensation for noncommutative perturbative quantum gravity might be highly non trivial. This is so because ghost condensation can generate a vector field with non-vanishing vacuum expectation value. This can break the Lorentz symmetry. The ghost condensation is also expected to modify the infrared behavior of the off-diagonal ghost propagator, while contributing to the vacuum energy density.

It will also be interesting to generalise the results of this paper to general curved spacetime. The generalisation to arbitrary spacetimes might not be simple as it is still not completely clear how BRST symmetry will work there. However, the generalisation to Anti-de Sitter spacetime might be straightforward because the ghost fields are expected to be well behaved in Anti-de Sitter spacetime [40]. For de Sitter spacetime there are issues with BRST invariance of the theory due to the infrared divergence of the ghost propagators [40]. Thus the generalisation of this work to de Sitter spacetime will not be straightforward.

It will also be useful to analyse the shift symmetry for these noncommutative theories in the anti-field formalism and express the result in superspace formalism. As the effect of shift symmetry in commutative field theories with higher derivatives has already been analysed [41], it will be interesting to generalise those results to noncommutative field theories including noncommutative perturbative quantum gravity.

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